Uncovering thick and thin strings via renormalons.

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Abstract

The talk is about the power corrections in QCD. Renormalons, both infrared and ultraviolet, provide with a kind of a kinematical framework to fix exponents of the leading power corrections to various observables. Any viable dynamical framework is to reproduce this pattern of the power corrections. In this mini-review, we emphasize that a simple dynamical framework satisfying this requirement may well be provided by the strings inherent to the Abelian Higgs model. This model comes naturally into consideration within the $U(1)$ projection of QCD which makes explicit the dual-superconductor model of the confinement. At large distances (compared to Λ_{QCD}^{-1}) there are Abrikosov-Nielsen-Olesen strings which develop a characteristic non-perturbative transverse size of order Λ_{QCD}^{-1} and match in this way the infrared renormalons. At short distances there are no ANO strings but there still exist dynamical manifestations of the stringy topological condition that (external) quarks are connected by a mathematically thin line along which the vacuum is trivial. These manifestations may match ultraviolet renormalons.

1 Outline of the review

Power corrections to the parton model is an actual topic in QCD. Any comprehensive review of it would ask for much more time in the oral presentation and space in the written version than is allocated in our case now. Moreover, there are quite a few reviews already available (see, e.g., [1,2,3,4,5]). Instead we would rather like to confine ourselves to a few background remarks and a mini-review of some of the latest developments. Thus, this written version of the Talk emphasizes mostly power corrections associated with short-distance non-perturbative physics, following the original papers [8,9]. Also, we emphasize that the classical solutions to the Abelien Higgs model may provide a unifying dynamical framework to reproduce the basic results of the renormalon calculus of the power corrections. The connection of this model to QCD is that the AHM realized the dual-superconductor model of the confinement [10]. The Abrikosov-Nielsen-Olesen strings are reproducing then the generic predictions of the infrared renormalons. Moreover the short-distance behaviour of heavy quark potential potential may be closely related to ultraviolet renormalons [11]. Within the Abelian Higgs model the potential at short distances appears

to posess a linear correction [8] which does match the ultraviolet renormalons. These key remarks are embedded in the Talk into a general picture of power corrections as revealed by the renormalons.

2 Renormalons

Renormalons is a simple perturbative device to establish limitations of the perturbative approach to QCD (for reviews and further references see, e.g., [1,2,4]). In more detail, one considers an observable O and calculates it perturbatively as a series in $\alpha_s(Q^2)$:

$$O = O_{parton\ model}\left(1+\sum_{n=1}^{\infty} a_n(\alpha_s(Q^2))^n\right), \tag{1}$$

where Q is a large mass scale characteristic for the process considered, like total energy in case of e^+e^--annihilation into hadrons.

The expansion coefficients a_n are calculated then in the single-renormalon-chain approximation, i.e. in terms of graphs with many bubble insertions into a gluon line. Then it is easy to see that the expansion coefficients grow factorially with n at large n:

$$a_n \ \sim \ n!\left(\frac{b_0}{o}\right)^n \tag{2}$$

where b_0 is the first coefficient in the expansion of the β-function, introduced here for convenience, and o is a number. This factorial growth of the coefficients is associated with integration over non-typical virtual momenta p^2, either $p^2 \ll Q^2$ (infrared renormalons) or $p^2 \gg Q^2$ (ultraviolet renormalons).

From direct computations one finds values of the constant o in Eq. (2). It turns out that the exponent o associated with the UV renormalons does not depend on the variable considered:

$$o_{UV} \ = \ -1. \tag{3}$$

As for the infrared renormalons the corresponding values of o do vary from case to case. The smallest value of o known so far appears in event shapes and, in particular, in case of the thrust:

$$o_{thrust} \ = \ \frac{1}{2}. \tag{4}$$

Since the series with $a_n \sim n!$ are at best asymptotical, the perturbative QCD cannot approximate the observable O to accuracy better than

$$\Delta \ \sim \ \left(\frac{\Lambda^2_{QCD}}{Q^2}\right)^{|o|}. \tag{5}$$

Reversing the statement, one may say that the renormalons signal presence of non-perturbative power corrections of order (5) which are to be added to the perturbative series to make the answer for the observable O unique.

Thus, single-renormalon chains provide with a unified way to determine for any observable the corresponding exponent o. It is important, however, that multi-renormalon uncertainties of the perturbative expansions (1) are in fact of the same order [12,1]. The reason is that there are two large parameters involved in the evaluation of the renormalon-type graphs. One of them is n itself and the other is $ln(p^2/Q^2)$ where p is the characteristic virtual momentum. Multi-renormalon chains lose powers of the log but win in powers of n. Since effectively $ln(p^2/Q^2) \sim n$ the two factors cancel each other. As for the value of o, it remains the same.

This simple observation implies that there is no model-independent way to relate power corrections to various observables even with the same exponent o. Indeed, to do so one has now to evaluate any number of the renormalon chains which is impractical.

3 Tube model and IR renormalons.

The sketchy presentation in the previous section was to convince the reader that the renormalons is rather a kinematical framework than a dynamical one. Indeed, even evaluating the renormalon contributions with all the coefficients involved does not allow to judge whether the power correction is big or small since the estimate (5) does not fix the overall coefficient. If the corrections are small numerically, on the other hand, it is very difficult to dig the power corrections out from the thick layer of pure perturbative terms. Also, the issue of multi-renormalon contributions makes the problem non-tractable by direct computations.

Thus, we need models. And at first sight, no particular mode is encoded in the values of the exponents o calculable via renormalons.

Still, one may argue that the tube model has good chances [13,1] to match the infrared renormalons. For the sake of definiteness we concentrate on the thrust T. The argument starts then with an explicit evaluation of the one-renormalon contribution to $< 1 - T >$. Moreover, to perform the calculation one notes that the role of the renormalon-type graphs is to replace constant α_s appearing in one-loop by the running coupling $\alpha_s(k_\perp^2)$. Indeed, this is the function of bubble insertions into the gluon line, to clarify the argument of the running coupling. Thus, one integrates over $k_\perp$ of the gluon at the last step and separates the infrared sensitive piece:

$$< 1 - T >_{1/Q} \approx \frac{2C_F}{\pi Q} \int_0^{\sim Q} dk_\perp \alpha_s(k_\perp^2) \tag{6}$$

where the integral is understood in such a way that only contribution of the Landau pole is kept and $2Q$ is the total CM energy.

The message brought by the Eq.(6) is quite clear. Namely, we should separate the contribution of soft gluons with $k_\perp \sim \Lambda_{QCD}$, reserve for an effective coupling of order unit for the emission of such gluons, and declare this piece to be a parameterization of non-perturbative contributions to the thrust. The most important observation is that the leading correction is of order Λ_{QCD}/Q.

To establish a connection of this derivation to the tube model (see, e.g., [14]) one proceeds as follows. First, for two-jet events the thrust is related to (heavy- and light-) jet masses:

$$< 1 - T >_{two-jet} \;=\; \frac{M_h^2 + M_l^2}{4Q^2} \tag{7}$$

Moreover,

$$\left(\frac{M_{jet}^2}{Q^2}\right)_{non-pert} \approx \frac{2\langle k_\perp\rangle}{Q} \tag{8}$$

where $\langle k_\perp\rangle \sim 0.5GeV$ is the non-perturbative transverse momentum of the hadrons inside the jet introduced by the model.

Clearly the two equations (8) and (6) refer to similar mechanisms of generating the power corrections through introduction of an intrinsic $\langle k_\perp\rangle \sim \Lambda_{QCD}$. It is no surprise then that renormalons reproduce, for example, the tube-model relation between the $1/Q$ corrections to the thrust and the C parameter:

$$\langle 1 - T\rangle_{1/Q} \;=\; \frac{2}{3\pi}\langle C\rangle_{1/Q}. \tag{9}$$

Moreover, within the tube model one predicts:

$$\left(\frac{\langle M_h^2\rangle}{Q^2}\right)_{1/Q} \approx \left(\frac{\langle M_l^2\rangle}{Q^2}\right)_{1/Q}. \tag{10}$$

The experimental data [15] agree with both (9) and (10) although with different accuracy.

Derivation of the trivially-looking Eq. (10) within some of the renormalon-based approaches proved not so simple. The point is that the standard renormalon technique which results in Eq. (6) implies at first sight that the renormalon contributions are a small fraction of the ordinary perturbative graphs. Thus, if the thrust is calculated to the order $\alpha_s(Q^2)$ then, one could argue, non-perturbative contribution to the jet masses can be counted also only once. Then the prediction would be that one of the jets has no nonperturbative contribution to its mass at all and is doomed to have a strictly vanishing mass which may not be true experimentally of course. Thus, one is led to assume that two-loop nonperturbative pieces should be kept track of even if the perturbative $(\alpha_s(Q^2))^2$ correction is neglected. This phenomenon can be dubbed as an enhancement of nonperturbative corrections [1].

The prediction (10) is natural within the universality picture [13,16] which

keeps terms which contribute perturbatively in differential distributions of the variables as $\alpha_s^m ln^k Q$ and continues such terms to the infrared region (where, in general, they do not dominate any longer). On the other hand, it contradicts the large N_f- or naive-nonabelianization schemes (for review and references see, e.g., [3,4]) which are tailored to keep only one-renormalon chains. The dispersive approach to the coupling (for a review and further references see [5]) in its original form [17] also results in $M_l^2 = 0$. However, more recently it was elevated to a two-loop level [18] and produces now predictions similar to the model of Refs. [13,16]. It is worth emphasizing, however, that the renormalon-based frameworks on the two-loop level do not reduce automatically to the tube model since in higher orders the soft gluon emission includes non-factorizable contributions as well [19].

Infrared renormalons were applied also to power corrections in DIS [20] and to shape variable distributions [21]. In these cases the predictions are richer since the power corrections depend on an extra variable, like the Bjorken x in case of DIS. The renormalon-induced power corrections to the structure functions $F_2(x,Q^2)$ tend to reproduce the gross features of the tube model [22]. In case of the structure functions $F_L(x,Q^2)$ the renormalon predictions appear to depend on details of introducing the infrared cut off [23], which is probably due to the fact that F_L vanishes in the parton-model approximation.

To summarize, both the tube model with intrinsic $< k_\perp >\neq 0$ and infrared renormalons predict the same values of the exponent o. As for relations between power corrections to various observables, the renormalon-based predictions do not contradict the tube model but provide, generally speaking, with a more general and vague framework.

4 Ultraviolet renormalons

As is mentioned above, the leading ultraviolet renormalon brings perturbative expansions of the form:

$$\left(\sum_n a_n\alpha_s^n(Q^2)\right)_{UV} \sim \sum_n n!(-b_0)^n\alpha_s^n(Q^2). \tag{11}$$

Because of the sign oscillations it is quite common to apply the Borel summation. The summation amounts to replacing the growing branch of the products $|a_n\alpha_s^n|$ by its integral representation:

$$\sum_{N_{cr}}^{\infty} n!(-b_0)^n\alpha_s^n \rightarrow \int \frac{(\alpha_s b_0 t)^{N_{cr}} exp(-t)dt}{1+\alpha_s b_0 t} \tag{12}$$

where $N_{cr} = 1/b_0\alpha_s$ is the value of n for which the absolute value of the terms in the series reaches its minimum. The right-hand side is readily seen to be of

order:

$$\frac{1}{2}\left(a_n\alpha_s^n(Q^2)\right)_{n=N_{cr}} \sim \frac{\Lambda^2_{QCD}}{Q^2}. \tag{13}$$

It is amusing to observe that this correction comes from huge virtual momenta of order $p^2 \sim Q^2 \cdot exp(N_{cr}) \sim Q^4/\Lambda^2_{QCD}$.

Although at first sight the Borel summation may look an arbitrary procedure it can be substantiated in the following way [24]. Instead of expanding in $\alpha_s(Q^2)$ as in Eq. (1) one could expand in $\alpha_s(\mu^2)$. Then the uncertainty of the perturbative expansions Δ (see Eq. (5)) does not stay invariant for a sign-alternating series but satisfies instead:

$$\Delta(\alpha_s(\mu^2)) \; = \; \Delta(\alpha_s(Q^2))\frac{Q^4}{\mu^4} \tag{14}$$

and can be made therefore arbitrary small by choosing μ^2 large enough. It seems obvious, furthermore, that the series in $\alpha_s(\mu^2)$ converges to the Borel sum of the series in $\alpha_s(Q^2)$.

Note that accepting the Borel summation does not imply elimination of the Λ^2_{QCD}/Q^2 corrections due to the UV renormalons. They are still generated through the summation procedure (12). Moreover, since the multi-renormalon contributions are not suppressed [12] there are no practical ways to evaluate the Λ^2_{QCD}/Q^2 terms by applying the Borel summation alone. From this general point of view there is no much difference from the case of IR renormalons. Numerically, of course, different corrections can be of very different scales.

To probe the Λ^2_{QCD}/Q^2 corrections of an UV origin we should consider quantities which do not receive similar corrections from the infrared. For example, the DIS would be a wrong choice since IR renormalons already induce Λ^2_{QCD}/Q^2 corrections. Thus, the central object to study UV renormalons [25] are the vacuum correlators of the currents j with various quantum numbers:

$$\Pi_j(Q^2) \; = \; i\int exp(iqx)\langle 0|T\{j(x),j(0)\}|0\rangle, \tag{15}$$

where $q^2 \equiv -Q^2$ and we suppressed the Lorentz indices and assumed that the currents are normalized to have zero anomalous dimension. Moreover, to get rid of the UV divergences one studies usually $\Pi(M^2)$ where [26]

$$\Pi_j(M^2) \; \equiv \; \frac{Q^{2n}}{(n-1)!}\left(\frac{-d}{dQ^2}\right)^n \Pi_j(Q^2) \tag{16}$$

in the limit where both Q^2 and n tend to infinity so that their ratio $M^2 \equiv Q^2/n$ remains finite. According to the standard OPE:

$$\Pi_j(M^2) \approx (parton\ model) \cdot \tag{17}$$

$$\cdot\left(1+\frac{a_j}{lnM^2/\Lambda^2_{QCD}}+\frac{c_j}{M^4}+O((lnM^2)^{-2}M^{-6})\right)$$

where the constants a_j, c_j depend on the channel, i.e. on the quantum numbers of the current j. The terms of order $1/lnM^2$ and M^{-4} are associated with the first perturbative correction and the gluon condensate, respectively.

A salient feature of Eq. (18) is the absence of Λ^2_{QCD}/Q^2 terms which is due to the fact that there are no gauge invariant operators of dimension d=2 which could have vacuum-to-vacuum matrix elements. Now, accounting for the UV renormalons brings Λ^2_{QCD}/Q^2 terms which go beyond the standard OPE:

$$\Pi_j(M^2) \approx (parton\ model) \cdot \tag{18}$$
$$\cdot \left(1 + \frac{a_j}{lnM^2/\Lambda^2_{QCD}} + \frac{b_j}{M^2} + \frac{c_j}{M^4} + ...\right).$$

In section 7 we review briefly a model to evaluate the coefficients b_j in various channels.

Another physical quantity which can be used to isolate the Λ^2_{QCD}/Q^2 corrections of the UV origin is the heavy quark potential $V(r)$ at short distances [11]. Indeed, it is obvious from dimensional considerations that the leading power correction to the perturbative potential at short distances r is now linear in r:

$$\lim_{r\to 0} V(r) = -\frac{4\alpha_s(r)}{3r} + \sigma r \qquad (non-perturbative\ 1/Q^2\ corrections). \tag{19}$$

On the other hand, within the so to say standard QCD the leading power correction at *short* distances is of order r^2:

$$\lim_{r\to 0} V(r) = -\frac{4\alpha_s(r)}{3r} + cr^2 \qquad (standard\ QCD). \tag{20}$$

This conclusion is based solely on the assumption that the nonperturbative fluctuations in QCD are of large scale, $\sim \Lambda^{-1}_{QCD}$ (for references and further explanations see [11]). Thus, the introduction of the linear correction to the potential through the new Λ^2_{QCD}/Q^2 terms from the ultraviolet assumes small-size nonperturbative field configurations. A particular picture of the vacuum properties which results in this effect is described in Ref [11].

5 ANO strings.

We will consider now the Abelian Higgs model (AHM) describing interactions of a $U(1)$ gauge field A_μ with a charged scalar field Φ in the Higgs phase when the scalar field condenses. The model is defined by its action :

$$S = \int d^4x \left\{ \frac{1}{4e^2} F^2_{\mu\nu} + \frac{1}{2}|(\partial - iA)\Phi|^2 + \frac{1}{4}\lambda(|\Phi|^2 - \eta^2)^2 \right\} \tag{21}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and the complex scalar field Φ carries electric charge. The physical vector and scalar particles are massive, $m^2_V = e^2\eta^2, m^2_H = 2\lambda\eta^2$ and for the sake of definiteness we assume $m_H > m_V$.

The model (21) is famous to provide with a relativistic analog of the Landau-Ginsburg theory of superconductivity. Namely, if one introduces a monopole-antimonopole pair as an external probe its static potential $V(r)$ grows linearly with the distance r at large r:

$$lim_{r\rightarrow\infty}V(r) ~=~ kr. \qquad (22)$$

This property is crucial for the dual-superconductor model of confinement [10]. Within this picture, one thinks about an external $\bar{Q}Q$ pair in the environment of a condensed scalar field which carries a non-zero monopole charge. Up to the change of the notations to the dual ones, the physics is the same as described by (21) and for the sake of definiteness we shall not change the notations, i.e. will be considering interaction of a monopole-antimonopole pair within the AHM.

The relevance of the AHM to QCD is most obvious in the $U(1)$ projection of QCD [27] when the diagonal gluons are treated as $U(1)$, i.e. photonic, gauge fields while the other (non-abeliean) components play the role of charged vector fields. Moreover, there exist detailed numerical simulations on the lattice which confirm the dual-superconductor picture of the confinement (for review and references see [28]).

The growth of the potential at large distances, i.e. at $r \gg \Lambda^{-1}_{QCD}$ is well understood in terms of formation of the Abrikosov-Nielsen-Olesen (ANO) strings (for review and further references see, e.g., [29]). The ANO strings [30] are solutions to the classical equations of motion corresponding to the action (21) with cylindrical symmetry and carrying a (quantized) magnetic flux $\Phi = 2\pi/e$. The magnetic field is distributed within the string as:

$$|H| = \frac{m_V^2}{e}K_0(m_Vr) \rightarrow_{r\rightarrow\infty} \eta\sqrt{\frac{\pi\eta}{2er}}e^{-m_Vr} \qquad (23)$$

where r is the distance in the transverse direction. As for the scalar, or Higgs field it disappears on the axis,

$$\lim_{r\rightarrow 0}|\Phi(r)| ~=~ 0 \qquad (24)$$

and approaches its vacuum value at large r:

$$\lim_{r\rightarrow\infty}|\Phi(r)| ~=~ \eta + (const)~e^{-m_Hr}. \qquad (25)$$

A string may either be closed or end up with monopoles. In the latter case the const k in Eq. (22) is the energy density per unit length of the ANO string,

$$k ~\sim~ \frac{\pi\eta^2}{2}ln\frac{m_H^2}{m_V^2} \qquad (26)$$

where we assumed that $m_H^2 \gg m_V^2$. If the latter inequality is not satisfied, the log is replaced by a smooth function of the ratio m_H/m_V. In case of QCD the formation of the ANO string is confirmed by the numerical simulations in the $U(1)$ projection and the space structure of the fields inside the string is indeed

well described by the classical equations of motion [31].

After this brief description of the ANO string and its relevance to QCD, we can go back to the power correction. The relation to the tube model is rather self-evident. Indeed, the ANO string introduces a characteristic transverse scale, $x_\perp \sim m_V^{-1} \sim (\Lambda_{QCD})^{-1}$. In the momentum space, we have $k_\perp \sim m_V$ and, as is discussed in the preceding sections, this is what is needed to match the power corrections predicted by the renormalons. Numerically [31], the value of m_V is quite large, $m_V \sim 1 GeV$ so that the corresponding power corrections are large, may be even too large to match the phenomenological estimates.

6 Heavy quark potential at short distances. Dirac strings.

Let us now discuss short-distance physics, looking for a possible link to the UV renormalons. At first sight, there is nothing left from the ANO string if we go to short distances, $r \ll m_V^{-1}$. And this is of course true. However, a closer analysis reveals that there exist in fact two other stringy objects which can survive at short distances [8,34]. Indeed, in the so called London limit, $m_H^2 \gg m_V^2$, or $\lambda \to \infty$ with η fixed, the size of the Higgs core of the ANO string is much smaller than the size of the magnetic field and one may consider the distances $m_V^{-1} \gg r \gg m_H^{-1}$. Then the question is, what happens to the linear piece (22) in the potential at such distances.

Even more intriguingly, there is an object in the theory which is nothing else but a mathematically thin line. This is the line connecting the monopole and anti-monopole singled out through vanishing of the scalar field Φ along this line. The existence of such a line connecting the monopoles can be considered as a topological condition: the boundary of the world sheet with $\Phi \equiv 0$ can be nothing else but world-trajectories of monopoles [27]. However, the language of the Dirac string [32] may provide with a better insight. The existence of the Dirac string follows, as usual, from the magnetic flux conservation.

The possibility of a dynamical manifestation of the Dirac string stems from the fact that it cannot coexist with $\Phi \neq 0$ so that Φ vanishes along the string. Indeed, the self-energy of the Dirac string, is normalized to be zero in the perturbative vacuum. To justify this assumption one can invoke duality and ask for equality of the self-energies of electric and magnetic charges. However, if the Dirac string would be embedded into a vacuum with $< \Phi > \neq 0$ then its energy would jump to infinity because there is a term $\sim \Phi^2 A_\mu^2$ in the Hamiltonian and $A_\mu^2 \to \infty$ for a mathematically thin Dirac string. Hence, $\Phi = 0$ along the string. One may say that the Dirac strings always rest on the perturbative vacuum which is defined as a vacuum state obeying the duality principle. Therefore, even in the limit $r \to 0$ there is a deep well in the profile

of the Higgs field Φ. This might cost energy which is linear with r even at small r.

Consider first the London limit. It can be studied analytically [34]. The result is that the answer for the potential is the same as if we had an ANO string fully developed [34]:

$$V(r) = -\frac{\pi}{e^2}\frac{e^{-m_V r}}{r} + \tag{27}$$
$$+\frac{\pi m_V^2}{2e^2}\left(r ln\frac{m_H^2}{m_V^2} - \frac{2}{m_V} + \int_0^\infty dk_\perp^2 \frac{e^{-r\sqrt{k_\perp^2+m_V^2}}}{(k_\perp^2+m_V^2)^{3/2}}\right).$$

Note that the linear correction to the potential persists despite of the fact that the distances considered are much smaller that the transverse size of the magnetic field within the ANO string, Naively, one would expect large edge effects which turn not to be there, however. One can consider this as an example of a dual description. Namely, in one language the ANO string is bulky, $x_\perp \sim m_V^{-1}$ while in the dual description it is entirely characterized by position of its center, $\Phi = 0$.

If we make the next step and consider $r \ll m_V^{-1}, m_H^{-1}$ then nothing is left from the original ANO strings. The field configuration is close to that of a magnetic dipole. Nevertheless, we still have a Dirac string (see above) which is manifested through a boundary condition $\Phi = 0$ along the straight line connecting the monopoles imposed on the solutions of the classical equations of motion. The potential at short distances is dominated by a Coulomb-like contribution. At intermediate distances the potential can be still found by solving the equations of motion numerically [33].

However, until very recently [8] there were no dedicated studies of the power correction to the Coulomb-like potential at short distances. The result of this recent study is that the linear correction to the potential still persists at $r \to 0$ (see Eq. (19)). Moreover, for $m_V \approx m_H$ the result for the linear correction at short distances is especially simple:

$$\sigma \approx k \tag{28}$$

where σr gives the linear correction at *short* distances (see Eq. (19)) while kr refers to the linear potential at *large* distances, see Eq. (22). Note that Eq. (28) approximately holds in spite of a complete change of the dynamical mechanism for the linear piece of the potential, which is the change from the ANO to Dirac strings and is true only for $m_V \approx m_H$.

It is amusing that the numerical simulations of the real QCD also give $k \approx \sigma$ [35] and (separately) $m_V \approx m_H$ [31] down to the distances studied so far. Moreover, the fact the slope k is not changing somehow masks the fact that the linear term σr at short distances is in contradiction with the standard

QCD, see Eq. (20). Thus, if no new short-distance non-perturbative effects are introduced the persistence of the relation (28) in the lattice measurements is to be attributed either to not-yet-small-enough distances r (the measurements extend down to $r \simeq 0.1 fm$, though) or to large error bars which do could have confused the fact that the linear piece actually vanishes.

On the other hand, a non-vanishing linear correction to the potential at $r \to 0$ is predicted within the classical approximation to the Abelian Higgs model [8]. Moreover, it is a direct dynamical manifestation of the Dirac string, or of the topological condition that the vacuum is trivial along a line connecting the monopoles [8]. If the AHM is to imitate QCD, this is for the first time that there a dynamical mechanism proposed which incorporates naturally the results of the measurements [35] of the potential on the lattice. Since in the $U(1)$ projection of QCD [27] the monopoles appear as singular, or point-like objects [36] there are good chances, to my mind, that the $U(1)$ description outlined above is a valid approximation to short distances. This question has not been studied in any detail so far, however.

Coming back to the renormalons, the linear correction to the $\bar{Q}Q$ potential at short distances would perfectly match, in the language of the preceding sections, the UV renormalons.

7 Short-distance tachyonic gluon mass.

The linear correction to the potential at short distances discussed in the previous section gives an example of a $1/Q^2$ correction of UV origin. It would be interesting to relate this correction to the $1/M^2$ terms in the current correlators (19). At this moment, however, a small-size-string correction to the current correlators cannot be evaluated from first principles. Instead we will review briefly in this section a simplified phenomenology in terms of a tachyonic gluon mass which is assumed to mimic the short-distance nonperturbative effects [9].

To motivate this, rather drastic assumption we simply notice that the linear term in the potential at short distances (see Eq. (19)) can be imitated [7] by the Yukawa potential with a gluon mass λ:

$$\frac{4\alpha_s}{6}\lambda^2 \ \sim \ - \ \sigma. \tag{29}$$

The tachyonic sign for the λ^2 arises because of the positive string tension of the small-size strings found in the preceding section. This notion of the short-distance gluon mass can be consistently used at one-loop level as well without running into a conflict with the gauge invariance. This observation is crucial to extend the phenomenology from the short-distance potential to the correlators (19).

Concentrating therefore on the phenomenology of these correlators we notice first that the basic quantity here is the scale M^2_{crit} at which the parton model gets violated by the power corrections in various channels [37]. More specifically, M^2_{crit} is defined as the value of M^2 at which the power corrections become, say, 10% from the unit. The meaning of M^2_{crit} is that at lower M^2 the power corrections blow up.

In the ρ-channel, M^2_{crit} is controlled by the M^{-4} term and:

$$M^2_{crit}(\rho - channel) ~\sim~ 0.6~GeV^2 \tag{30}$$

where the numerical value of M^2_{crit} is fixed by the magnitude of the gluon condensate $< \alpha_s(G^a_{\mu\nu})^2 >$. Moreover, it agrees well with independent evaluation of M^2_{crit} from the experimental data on the e^+e^- annihilation [26].

If one proceeds to other channels, in particular to the π-channel and to the $0^{\pm}$-gluonium channels, nothing special happens to M^2_{crit} associated with the gluon condensate. However, it was determined from independent arguments that the actual values of M^2_{crit} do vary considerably in these channels [37]:

$$M^2_{crit}(\pi - channel) ~\geq~ 1.8~GeV^2 \tag{31}$$

$$M^2_{crit}(0^{\pm} - gluonium~channel) ~\geq~ 15~GeV^2. \tag{32}$$

These lower bounds on M^2_{crit} are obtained from the values of f_π and of the quark masses in the pion channel, and from a low-energy theorem in the gluonic channel. Such values of M^2_{crit} cannot be reconciled with the assumption that the gluon condensates controls M^2_{crit} in all the channels.

Now, that we have introduced a short-distance gluon tachyonic mass, the coefficients b_j are calculable in terms of λ^2 [9]:

$$b_\pi \approx 4b_\rho = \frac{4\alpha_s}{3\pi}c_{gluonium} = -\frac{4\alpha_s}{\pi}\lambda^2. \tag{33}$$

Phenomenologically, in the ρ-channel there are severe restrictions [38] on the new term b_ρ/M^2:

$$b_\rho ~\approx~ (0.03 - .07)~GeV^2. \tag{34}$$

Remarkably enough, the sign of b_ρ does correspond to a tachyonic gluon mass. Moreover, when interpreted in terms of λ^2 the constraint (34) does allow for a large λ^2, say, $\lambda^2 = -0.5GeV^2$.

Another crucial test is the effect of $\lambda^2 \neq 0$ on the value of $\alpha_s(M^2_\tau)$ as determined from the width R_τ of the leptonic τ-decays. Indeed, the running coupling is well known independently. It turns out that $\lambda^2 \approx 0.5GeV^2$ brings $\alpha_s(M^2_\tau)$ down by about 10%. The sign of the change is again the right one to improve the agreement with $\alpha_s(m^2_Z)$ and the absolute value of the change is within experimental uncertainties.

As for for the π-channel one finds now a new value of M^2_{crit} associated with $\lambda^2 \neq 0$:

$$M^2_{crit}(\pi - channel) \approx ~4 \cdot M^2_{crit}(\rho - channel) \tag{35}$$

which fits nicely the Eqs. (30) and (31) above. Moreover, the sign of the correction in the π-channel is what is needed for phenomenology [37]. This can be considered as another crucial test of the tachyonic sign of the λ^2. Fixing the value of c_π to bring the theoretical $\Pi_\pi(M^2)$ into agreement with the phenomenological input one gets

$$\lambda^2 \approx -0.5\, GeV^2. \tag{36}$$

Finally, we can determine the new value of M^2_{crit} in the scalar-gluonium channel and it turns to be what is needed for the phenomenology, see Eq (32). Thus, qualitatively the phenomenology with a tachyonic gluon mass which is quite large numerically stands well to a few highly nontrivial tests.

It is worth emphasizing that the λ^2 terms represent nonperturbative physics and limit in this sense the range of applicability of pure perturbative calculations. This nonperturbative piece may well be much larger than some of the perturbative corrections which are calculable and calculated nowadays.

Further crucial tests of the model with the tachyonic gluon mass could be furnished with measurements of various correlators $\Pi_j(M^2)$ on the lattice [9].

8 Conclusions

We discussed briefly the status of the power corrections associated both with IR and UV regions. We argued that the Abrikosov-Nielsen-Olesen and Dirac strings of the $U(1)$ projection of QCD appear to match renormalons, infrared and ultraviolet respectively. Phenomenologically, there is room for a new, relatively large Λ^2_{QCD}/Q^2 correction coming from the ultraviolet. If confirmed, such a correction would be of great interest.

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